

1. For each the following functions: Identify all asymptotes (if applicable); find all local and absolute extrema; find the points of inflection; sketch the graph and mark the special points on the graph.

(a) (5 points) $f(x) = \cos(2x)$ on the interval $[-2\pi, 2\pi]$.

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(b) (5 points) $g(x) = \frac{2x}{x^2-4}$ on $(-\infty, \infty)$.

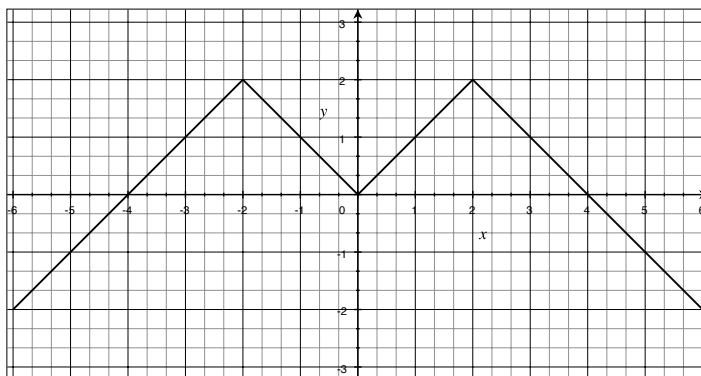
2. (10 points) You are to construct an open rectangular box with a square base and a volume of $36 \text{ (ft}^3\text{)}$. If material for the bottom costs \$8 per $\text{(ft}^2\text{)}$ and material for the sides costs \$5 per $\text{(ft}^2\text{)}$, what dimensions will result in the least expensive box? What is the minimum cost?

3. (10 points) The relationship between resistance R , voltage V and current I in a circuit is called the Ohm's Law:

$$I = \frac{V}{R}.$$

When $R = 75$ (ohm) and $V = 110$ (volt), R is decreasing at the rate of 1 (ohm/sec), and I is increasing at the rate of 0.5 (amp/sec). At what rate is V changing then? (Hint: write V in terms of R and I . Think of each as a function of t and differentiate .)

4. (4 points) Consider a function $y = f(x)$ represented by the graph below. Let's define $F(x) = \int_0^x f(t) dt$.



- (a) (3 points) Find critical points of F .
- (b) (3 points) Find points of inflection of F .
- (c) (3 points) Find all local and global extrema of F .
- (d) (3 points) Sketch $y = F(x)$ and mark all the special points.

5. Find the derivatives and the region of differentiability of the following functions.

(a) (3 points) $y = \sin(x)\sqrt{x}$.

Region of differentiability:

(b) (3 points) $y = \frac{\cos(x)}{1+\cos(x)}$.

Region of differentiability:

(c) (3 points) $y = \int_1^x \frac{1}{s} ds$.

Region of differentiability:

(d) (3 points) $y = \int_1^{\cos x} |t| dt$.

Region of differentiability:

6. Find the following indefinite integrals:

(a) (3 points) $\int \cos(2x) \, dx.$

(b) (3 points) $\int (x^{16} + \sqrt[16]{x} + \frac{1}{x^{16}}) \, dx.$

(c) (3 points) $\int \pi \, dx.$

(d) (3 points) $\int \csc \frac{x}{2} \cot \frac{x}{2} \, dx.$

7. (a) (2 points) State the Fundamental Theorem of Calculus part II.

- (b) (5 points) Use FTC II to find the area between the graph of the following function and the x-axis over the indicated interval. Make a sketch.

$$f(x) = |x^3|, \text{ over the interval } [-1, 3].$$

- (c) (5 points) Using mid-point rule with 4 equal sub-intervals, approximate the area in part (b) above. What's the error in this approximation?

8. At time t , the position of a body moving along the s -axis is $s(t)$. Suppose $s(0) = 0$ and It's initial velocity is $v(0) = -10$. The body is experiencing an acceleration of $a(t) = 6t - 6$.

(a) (3 points) Find the body's velocity as a function of time: $v(t)$.

(b) (3 points) Find the body's position function $s(t)$.

(c) (2 points) Find the average velocity over the time interval $[2, 4]$ using the displacement over time formula.

- (d) (3 points) State the Mean-Value theorem for derivatives and explain how it can be used to conclude that instantaneous velocity $v(t)$ must have been equal to zero at some point in the time interval $[2, 4]$.

- (e) (3 points) Find the average of the velocity function over the time interval $[2, 4]$ using the Definite Integral definition of averages.

- (f) (1 point) How is this notion connected with the average velocity based on the distance function. (Hint: use the Fundamental Theorem of Calculus.)



- (g) (7 points) Sketch the graphs of the acceleration, the velocity and the distance functions. Identify zeros, critical points, inflection points local and global minima and maxima. What's happening to the body at the critical points of the position function?

Extra CreditExtra CreditExtra Credit

9. (Extra Credit points) (a) (2 points) Set up the Riemann Sum approximation to the integral $\int_1^2 \frac{1}{\sqrt{x}} dx$ with n intervals and using right end-point of each interval as c_k .

- (b) (2 points) Use $n = 10$ to compute an approximation to $\int_1^2 \frac{1}{\sqrt{x}} dx$ using right end-point of each interval as c_k . Is this value an over-estimation or an under-estimation of the integral? Explain why. (Hint: you should sketch the function and the boxes for this approximation.)

- (c) (2 points) Compute the definite integral $\int_1^2 \frac{1}{\sqrt{x}} dx$ by using the Fundamental Theorem of Calculus.

- (d) (2 points) How are your answers to (a) and (c) related?

- (e) (2 points) Can you compute the following limit? Why or why not?

$$\lim_{n \rightarrow \infty} \frac{1}{n} \left(\frac{1}{\sqrt{1+1/n}} + \frac{1}{\sqrt{1+2/n}} + \frac{1}{\sqrt{1+3/n}} + \dots + \frac{1}{\sqrt{1+k/n}} + \dots + \frac{1}{\sqrt{1+n/n}} \right).$$